Experimental validation of analytic formulas for the statistical uncertainty in the Feynman–α method

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A B S T R A C T
Noise experiments, and the Feynman–α method in particular, are considered a standard in-pile experiment, often used in zero power reactors in a sub critical configuration to estimate the reactivity level. In the Feynman–α method, the variance to mean ratio of event triggered neutron count is fitted on the so called Feynman-Y function, resulting with an experimental estimation of the α decay mode.

In a recent study, analytic formulas for the expected statistical uncertainty in the Feynman–α method were derived, using the backward stochastic transport equation. The outline of the present study is to experimentally validate these analytic formulas. The formulas were implemented on 4 different signals, obtained from the MINERVE reactor at CEA Cadarache. For each of the four measurements, the signal was truncated in increasing measurement lengths (ranging from 1 to 60 min), in order to observe the functional behavior of the statistical error through time. Then, the measured statistical error was compared with the analytic estimation.

Results indicate a very high correspondence between the expected statistical error and the measured one.

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1. Introduction

Noise experiments are considered a standard in-pile experiments, often used in zero power reactors in a sub critical configuration to estimate the reactivity level. In the Feynman–α method, originating in the seminal work of Feynman (originally introduced in Feynman (1945), and later revisited in Feynman (1956)) the variance to mean ratio of event triggered neutron count in gates of length $T$ (the Feynman-Y curve) is fitted on the so called Feynman-Y function, resulting with an experimental estimation of the α decay mode. The original formalism for the Feynman-Y function used a single decay mode, thus neglecting the effect of delayed neutrons (resulting, in practice, with a restriction on the duration of the gate $T$ Uhrig, 1970). Soon after, a multi-exponential model was constructed, allowing us to consider the effect of the delayed neutrons (Williams, 1974).

Although introduced over 60 years ago, the Feynman–α method still studied, both through theory and application, from a large number of contributors: Experimental results are reported on Szieberth et al. (2012), Kloosterman and Rugama, 2005, dos Santos et al., 2007 and Gilad et al., 2016 (to state a few), and from a theoretical point of view, generalizations of the Feynman-Y function to more elaborated settings is given in Malinovitch and Dubi (2015), Kitamura et al. (2005) and Ceder and Pazsit (2003) and many more.

As in any experimental method, results are never fully complete without an uncertainty estimation. In general, the term "uncertainty" relates to three elements: systematic errors, due to the model assumptions, numerical errors, due to the computation procedures, and finally, the statistical error, due to the statistical nature of neutron counting.

In a recent study (Dubi and Kolin, 2016), the statistical uncertainty on the reactivity estimation in the Feynman–α method was studied from an analytic point of view, resulting with an explicit formula for the statistical uncertainty in terms of the system parameters. The outline of the present study is to validate the results introduced (Dubi and Kolin, 2016) experimentally. The experimental validation was done on a set measurements taken at the MINERVE reactor (Perret et al., 2017), during a noise measurement campaign, held in Sep. 2014. Results indicate a very good correspondence between the experimental results and the analytic prediction.

The paper is arranged in the following manner: In Section 2 we give the necessary background on the Feynman–α method and describe the formulas introduced in Dubi and Kolin (2016) used...
in the present study. In Section 3 we describe the experimental setting, and describe the methodology used to verify the applicability of formulas. In Section 4 we introduce the experimental results and in Section 5 we conclude.

2. Theoretical background

2.1. The Feynman-\(\alpha\) method

In the Feynman-\(\alpha\) method we sample the variance to mean ratio of the number of detections in a gate of duration \(T\), and compare the results with the so called Feynman-Y function.

Technically, the variance to mean ratio can be evaluated in the following manner: the detection signal is divided into \(N\) consecutive gates of duration \(T\). Denoting by \(X_j\) the number of detections in the \(j\)th interval (\(1 \leq j \leq N\)), and denote:

\[
E(T) = \frac{1}{N} \sum_{j=1}^{N} X_j, \quad V(T) = \frac{1}{N-1} \sum_{j=1}^{N} (X_j - E)^2, \quad Y(T) = \frac{V(T)}{E(T)} - 1;
\]

(2.1)

The sample values of \(Y(T)\) will be referred to as the Feynman-Y curve or the Feynman-Y plot.

For the analytic expression of \(Y(T)\), to which the sampled data is fitted, two models commonly are used. The first model is the prompt reactivity model, given by Feynman (1945):

\[
Y(T) = Y_\alpha = 1 - \frac{1 - e^{-\alpha T}}{\alpha T}
\]

(2.2)

where \(\alpha = \frac{\rho - \beta}{2}\).

The model incorporating the delayed neutrons is a bit more complicated. The Feynman-Y function is given by:

\[
Y(T) = C \times \sum_{j=1}^{7} 2 A_j \frac{1 - e^{-\beta_j T}}{\beta_j T}
\]

(2.3)

where \(A_j\) are defined by the residues of the zero power transfer function

\[
H_0(\omega) = \frac{1 - i \omega \sum_{k=1}^{6} \frac{A_k}{\lambda_k + i \omega}}{i \omega \Lambda + \sum_{k=1}^{6} \frac{A_k}{\lambda_k + i \omega}} = \sum_{j=1}^{7} A_j
\]

(2.4)

and \(\beta_j\) are the (7 distinct) roots of the Inhour equation:

\[-\beta_j \left( A + \sum_{k=1}^{6} \frac{\beta_k}{\lambda_k - \beta_j} \right) - \rho = 0
\]

(2.5)

Although the Feynman-Y formula depends on 7 decay modes, since all the modes are connected through the Inhour equation, there are effectively only two fit parameters: \(\rho\) and \(C\). Since the Feynman-Y function can not be written directly in terms of \(\rho\), the fit must be done on an implicit function; while this is not as simple as fitting data to an explicit function, it is still very doable from a numeric point of view.

2.2. Analytic formulas for the statistical uncertainty in the Feynman-\(\alpha\) method

The main theme of the present study is the experimental validation of the analytic formulas introduced in Dubi and Kolin (2016). A full description of the construction and the implementation of the formulas is given in Dubi and Kolin (2016), but for sake of completeness, a short description is added.

It should be stated that the formulas were constructed under the prompt reactivity model. Thus, \textit{a priori}, their applicability to the delayed reactivity model is questionable.

Estimating the statistical uncertainty in the Feynman-\(\alpha\) method is done in two steps. First, the statistical uncertainty on the sampled values of \(Y(T)\), denoted by \(\delta Y(T)\), is estimated. Then the propagation of the error from the sampled values of the Feynman-Y curve to the final result- the decay constant \(\lambda\) or the reactivity \(\rho\)-is computed.

For the first, the statistical uncertainty of \(Y(T)\), as introduced in Dubi and Kolin (2016) is given by:

\[
\delta Y(T) = \left| \frac{M_2(T)}{M_1(T)^2} \right| + \frac{1}{N_T} \left| \frac{M_2(T) - M_1(T)}{M_1(T)} \right| + \frac{1}{N_T} \left| \frac{M_2(T) - M_1(T)}{M_1(T)} \right|
\]

(2.6)

where \(M_i\) (\(i = 1, 2, 3, 4\)) is the \(i\)th moment of the number of detection in an interval of duration \(T\), and \(N_T\) is the number of intervals sampled. The values for \(M_1, M_2, M_3, M_4\) can either sampled, or estimated through analytic formulas introduced in Dubi and Kolin (2016). Since the formulas are very lengthy, they will not be repeated here, and the reader is referred to Dubi and Kolin (2016).

Before we continue, some clarification is required regarding formula (2.6). Eq. (2.6) is not a formula for variance of the Feynman-Y value, but rather a geometric approximation of the uncertainty on \(Y\). In more details: The Feynman-Y may be explicitly written as a function of the first to moments of the count distribution by:

\[
Y(T) = Y(M_1(T), M_2(T)) = \frac{M_2(T) - M_1(T)}{M_1(T)} - 1
\]

A first (and somewhat natural) approach to quantify the uncertainty on \(Y(T)\) is to evaluate the variance of \(Y(T)\) directly. This was, for instance, the approach used in Cramer (1962) (see chapter 27/7), where a first order approximation for the variance of a general function of any two moment of a distribution is introduced (an implementation may be found in Berglof et al. (2011)). Clearly, since the two moments are sampled from the same distribution they are correlated, and thus the formalism introduced in Cramer (1962) depends on the covariance of the two moments. The approach taken in Dubi and Kolin (2016) is a bit different, and geometric by nature. Consider \(Y(T) = Y(M_1(T), M_2(T))\) as a general differential function of the first two moments. For a small increments in \(dM_1, dM_2\) in \(M_1, M_2\) (respectively), the increment in \(Y\) may be approximated by:

\[
dY = \frac{\partial Y}{\partial M_1} \, dM_1 + \frac{\partial Y}{\partial M_2} \, dM_2
\]

(2.7)

Eq. (2.7) has nothing to do with probability or moments. The fact that \(M_1, M_2\) are sampled values with a strong correlation between them is bound to effect the relation between \(dM_1\) and \(dM_2\) but will not affect the applicability of Eq. (2.7). Next, assume the statistical uncertainty on \(M_1\) and \(M_2\) are \(\sigma M_1, \sigma M_2\). This means that the "true" values of the moments are somewhere in the rectangular \([M_1 - \sigma_1, M_1 + \sigma_1] \times [M_2 - \sigma_2, M_2 + \sigma_2]\). If we assume that the uncertainty is small enough (with respect to \(M_1\) and \(M_2\), then the "true" value of \(Y\) is in the interval \([Y(M_1, M_2) - \delta Y, Y(M_1, M_2) + \delta Y]\) where \(\delta Y\) is given by:

\[
\delta Y = \left| \frac{\partial Y}{\partial M_1} \right| \sigma M_1 + \left| \frac{\partial Y}{\partial M_2} \right| \sigma M_2.
\]

(2.8)

Thus, \(\delta Y\) (as defined in Eq. (2.8)) serves as an estimation of the statistical uncertainty.

Eq. (2.6) is nothing more than a realization of equality (2.8) using the explicit formula of \(Y(T)\) (as a function of \(M_1\) and \(M_2\)), and taking the standard deviation of \(M_1\) and \(M_2\) as the uncertainty of \(M_1\) and \(M_2\). Notice, although Eq. (2.8) is not an explicit formula for the standard deviation, since the uncertainty on \(M_1\) and \(M_2\) are
taken as the standard deviation, we expect to see a correlation between the two.

This approach for error estimation and error propagation is fairly common: A similar approach for the error propagation (in a slightly different context) is discussed in Smith-Nelson et al. (2015).

Once the uncertainty on the Feynman-Y values is computed, we need to quantify how this uncertainty will propagate to the uncertainty the output of our measurement: the reactivity $\rho$ (or the decay coefficient $\gamma$). Assume the Feynman-Y function is evaluated at gates $T_1 < T_2 < \ldots < T_N$, once $\sigma_{Y(T_j)}$ is evaluated, the statistical uncertainty on $\alpha$ is given by Dubi and Kolin (2016):

$$\Delta \alpha = \frac{\sum_{j=1}^{N} \left[ \frac{1}{\sigma_{T_j}} \left( 1 - e^{-\frac{\alpha T_j}{\sigma_{T_j}}} \right) + \frac{1}{\sigma_{\alpha T_j}} \left( \frac{1}{\sigma_{\alpha T_j}} \right) \right]}{\sum_{j=1}^{N} \left[ \sigma_{T_j^2} \left( 1 - e^{-\frac{\alpha T_j}{\sigma_{T_j}}} \right) + \sigma_{\alpha T_j^2} \left( \frac{1}{\sigma_{\alpha T_j^2}} \right) \right]}$$

(2.9)

Finally, since the reactivity (in the prompt reactivity of model) is linear with $\alpha$, we have that

$$\Delta \rho = \Delta \alpha$$

### 2.3. System parameters for the implementing the formulas

As mentioned earlier, the moments $M_i$, $i = 1, 2, 3, 4$ can be evaluated through analytic formulas developed in Dubi and Kolin (2016). It should be mentioned that an alternative method for evaluating the first four moments is simply to sample them. However, this has two problems: First, sampling the moments is obvioulsly possible only after the experiment- thus totally useless in terms of planning. Second, sampling the fourth moment of a highly multiplying system is very likely to have a very big statistical uncertainty, since we expect the variance of the fourth moment to by proportional to $\frac{1}{(1-k)^4}$.

Implementation of the formulas for $M_i$, $i = 1, 2, 3, 4$ demand numeric values for the following parameters: the decay coefficient $\alpha$- which is estimated directly through the Feynman-$\alpha$ method, the first four factorial moments of the multiplicity distribution of the fissionable material- which can be found in the literature, and two additional parameters: the source rate $S$ and the detection efficiency $P_d$.

The last two, if no initial information is given, may be evaluated in the following manner: First, the coefficient of the prompt reactivity Feynman-Y function (formula (2.2)) is given by:

$$Y_\infty = \frac{D_1 P_D k_p}{D_1 (1 - k_p)^2}$$

Where $k_p$ is the prompt multiplication factor (which is directly estimated through the Feynman-$\alpha$ method) and $D_1, D_2$ are the first and second factorial moments of the multiplicity distribution of the fissionable material (which can be found in the literature). Since $Y_\infty$ is evaluated in the fit, $P_d$ can be computed by

$$P_d = \frac{Y_\infty D_1 (1 - k_p)^2}{k_p D_2}$$

(2.10)

Next, since the counts per second (cps) is given by $cps = \frac{S \times P_d}{T}$, the source rate may be computed by:

$$S = \frac{cps \times (1 - k)}{P_d} \times 1 $$

(2.11)

(notice, here $k$ is the total multiplication factor, including the delayed neutron fraction).

It should be clear that the error bar on the values obtained by these considerations is fairly large, estimated at 10%-20%. However, this uncertainty is acceptable for two reasons: First, the sensitivity of the formulas introduced in Dubi and Kolin (2016) to uncertainties on $S$ and $P_d$ is linear. Thus the 10%-20% will not be amplified. Then, the 10%-20% refer to the relative error on the error, which, with respect to the actual observable is very small (for instance, if the statistical uncertainty is 10%, then the uncertainty on $S$ and $P_d$ will result in a variance in the range of 8–12%).

### 3. Experimental setting

#### 3.1. Core setting

The experiments were held during Sep. 2014, on the MINERVE reactor Perret et al. (2017), using the MAESTRO core configuration. The MINERVE reactor, a part of the CEA Cadarche complex, is a water moderated ZPR. The experiment detailed here took place in the framework of a pile noise experiment using high sensitivity fission chambers (PHOTONIS CFUL-01 with 1 g of U-235) located in the graphite reflector next to the driver zone fuel elements.

Measurements were taken at two reactivity levels, each measured in two distinct detectors, resulting with 4 detection signal to be analyzed. Preliminary results of the noise campaign were published (Glad et al., 2016). The reported reactivity of the first configuration (Acq16) was $-270$ pcm and 120 pcm for the second (Acq19). Both reactivity values were obtained by a rod drop experiment. Each measurement was approximately 1.5 h.

As described, implementation of the formulas require numerical values for the detection efficiency ($P_d$) and the amplitude of the external source ($S$). Using the formulas introduced, the relevant data of all four signal is provided in Table 1 below:

We can see that indeed the is a spread of about 10–15% in the detection efficiency and the source. Notice, this does not indicate that the source changed between the different experiments (in fact, it did not), but rather a testament to the uncertainty on the method used to evaluate $S$ and $P_d$.

#### 3.2. Validation methodology

The most basic and natural strategy to validate any approximation for a statistical uncertainty is to repeat the measurement for a large number of times, and then compare the statistical spread between the different measurements and the approximation. Indeed, such a comparison will be introduced in Section 4.1.

However, this strategy is problematic to apply in the present context, since it simply too time consuming. For instance, estimating the spread of a 15 min. measurement, assuming at least 35 measurements are required, would demand at least 8 h of measurements.

Therefor, in the present study, a different strategy was used. The total measurement (of duration $T_{meas}$), was truncated repeatedly, each truncation reducing one minute from the measurement time, stopping at a one minute measurement. Then, for each truncated signal, the reactivity was evaluated using the Feynman-$\alpha$ method.

<table>
<thead>
<tr>
<th>Measurement indicator</th>
<th>Reactivity (pcm)</th>
<th>Detector efficiency</th>
<th>Source rate S [n/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acq16</td>
<td>-270</td>
<td>4.37 $\times 10^5$</td>
<td>2.22 $\times 10^6$</td>
</tr>
<tr>
<td>Acq19</td>
<td>-120</td>
<td>3.86 $\times 10^5$</td>
<td>2.03 $\times 10^6$</td>
</tr>
<tr>
<td>Acq19</td>
<td>-120</td>
<td>4.25 $\times 10^5$</td>
<td>2.01 $\times 10^6$</td>
</tr>
</tbody>
</table>
This procedure will create a graph of the evaluated reactivity \( \rho(T) \) vs. the measurement time \( T \) (1 \( \leq T \leq T_{\text{tot}} \)).

Finally, we denote by \( \rho_1 \) the estimated reactivity using the total measurement (which was approximately 1.5 h long), and set \( \delta_\rho(T) = (\rho(T) - \rho_1) \). Since all the reactivity values were obtained using the same experiment (using the same methodology) differing only in the measuring time, the values of \( \delta_\rho(T) \) are governed by the statistical uncertainty.

We can not expect that \( \delta_\rho(T) \) will be equal to the statistical uncertainty, but we do expect that the general trend of \( \delta_\rho(T) \) and the estimated statistical uncertainty will be similar. Clearly, for two values \( T_2 > T_1 \), \( \delta_\rho(T_1) \) and \( \delta_\rho(T_2) \) are correlated, since one measurement is included in the other. In particular, we expect that all the values of sign of \( \delta_\rho(T) \) (positive or negative) will be correlated. Still, we expect that the absolute value of \( \delta_\rho(T) \) will reduce in the same rate the statistical uncertainty is reduced.

In all the samplings of the Feynman-Y curve, data acquisition was done in time stamping mode in very high resolution of 25 nano seconds, and a detector dead time about 50 nano second. The count rate was under \( 10^5 \) in all detection signals, thus dead time losses and channel shifting errors were negligible, so no bunching technique was used. The average and variance were sampled between consecutive time windows (as described in Eq. (2.1)). To create the Feynman-Y curve, the mean and variance was sampled in 200 gates, exponentially distributed in the interval \( [10^{-3}, 1] \) s. For the prompt reactivity analysis, only gates smaller than 0.1 s were considered, resulting with a total of about 130 gates in the interval \( [10^{-3}, 1] \) s (see Fig. 3).

4. Experimental results

4.1. Reactivity distribution for short measurement time

A basic validation of the formulas introduced in Dubi and Kolin (2016) is to break the entire measurement in a large number of sub measurement, and compare the statistical spread between the sub measurements and the results using formulas 2.6 and 2.9. Since we need to collect enough data to make the experimental approximation viable, we have broken the entire measurement into 1 min. and 2 min. sub measurements, resulting with about 90 and 45 sub measurement (respectively). The distribution for the measured decay coefficient \( \alpha \) in measurement Acq19 (data recorded by detector 1) when broken to 1 and 2 min. sub measurements is shown in Figs. 1, 2 (respectively).
In term of the reactivity, the measured standard deviation was 103 pcm for 1 min. measurements and 71 pcm for 2 min. measurements (notice, the ration between the two is 1.45, about 2.5% biasing from the theoretical ratio $\sqrt{2}$). Using formulas 2.9 and 2.6, the estimated statistical uncertainty is 95 pcm and 67 pcm respectively. That is, the formulas predicted the statistical uncertainty within 7 pcm error bar. As explained, these results are surprisingly good, since the analytic estimation for the statistical uncertainty is not the standard deviation explicitly, but rather the linear first order approximation of the increment in terms of the standard deviation of $M_1$ and $M_2$.

4.2. Implementation for long measurement times on the prompt reactivity model

We now turn to the second methodology, aimed to validate the formulas for long measurement times.

Fig. 3 shows the Feynman-Y plots obtained from the first detector of measurement Acq19, for a 1 min. measurement, 8 min. measurement and a 15 min. measurement. Error bars are achieved by implementation of formula 2.7. As we can see (and fully expected), the spread of the Feynman-Y curve reduces as
the measurement time increases. Moreover, we can observe a
"drift" of the Feynman-Y curve (the drift is hard to notice in the
time scales relevant to the prompt reactivity measurement, but
nevertheless, it is there).

The total duration of each measurements was about $T_{tot} = 1.5$ h.

Fig. 4 shows the predicted statistical uncertainty for each of the
four measurements as a function of the measurement time. First
thing to notice is a $1/\sqrt{T}$ functional behavior, as expected in statisti-
cal error.

We can see that for very short measurements- a single minute-
a statistical uncertainty of 90–120 pcm is expected. Then, in the
first 12 min, the statistical uncertainty drops dramatically in about
80 pcm, and in the remaining time, the statistical error reduces by
a merely additional 20 pcm, which is approximately the numeric
uncertainty. Therefor, after the 12 min threshold, we expect to
see a more or less stable value of $\delta_p(T)$. Indeed, results of all four
measurements indicated that after the first 12 min, $\delta_p(T)$ was
stable, with very small fluctuations (which may also be attributed
to the numerical uncertainty).

The results are for the prompt reactivity model in a time range
of 1–12 min. shown in Fig. 5. The error bars in each measurement
indicates the $1\sigma$ approximation for the statistical uncertainty using
the formulas in Dubi and Kolin (2016).

Noticeable, there is a very good correspondence between the
expected values and the measurement values. It is hard quantify
the correspondence between the measured values of $\delta_p(T)$ and
the estimated uncertainty, since the distribution of reactivity is
not known. However, a reasonable quantification is to compare
the results with a normal distribution. In such case, out of the 52
point shown in Fig. 1, 40 are inside the $1\sigma$ envelope (76%). This
is a bit more than expected from the $1\sigma$ envelope (67%), but if
we reduce the error bar by 50%, the number of points inside the
$1\sigma$ envelope drops dramatically to 48%. Thus, the expected statisti-
cal uncertainty using the formulas in Dubi and Kolin (2016) form
a fairly tight approximation for the statistical error.

4.3. Implementation for long measurement times on the delayed
reactivity model

The formulas for the statistical uncertainty were derived under
the prompt reactivity model, treating the delayed neutron source
as a fixed Poissonic source. It has been argued in Dubi and Kolin
(2016), that since the prompt reactivity model may be viewed as
a first order approximation of the delayed neutron model achieved
by a linear approximation of the lnhour equation, the approxima-
tion for the statistical error should hold to sum extent.
To validate this suggestion, the exact same analysis was carried out, but now the reactivity was computed using the delayed reactivity model. Once again, after 12 min the approximation was very stable, and for short measurements, the results are shown in Fig. 6.

As we can see, in very short measurements (1–2 min), the actual deviation is considerably larger than the expected statistical uncertainty. However, once the measurements are longer than 5 min (which is the general case, when considering noise experiments) we once again see a very good correspondence between the measured deviation and the expected uncertainty. Therefore, we can conclude that the formulas introduced in Dubi and Kolin (2016) are validated for the delayed reactivity model as well.

To prevent miss understanding- we wish to emphasize that the fact that the statistical error in both methods in the same does not indicated that the two methods have the same accuracy: since the comparison of the reactivity was carried out against a measured value (and not fixed value) using different analysis methods, the methodology says nothing at all about the systematic error, only the statistical error! A full analysis of the results is out of the scope of the present study, but it has been established that delayed reactivity model shows better results (especially in marginally critical system).

5. Concluding remarks

The outline of the present study was the experimental validation of analytic formulas for the statistical uncertainty in the Feynman-\(\alpha\) method.

The validation was done over 4 different signal, recorded by two detectors in two different sub critical configurations. The experiments were held at the MINERVE reactor in Cadarache.

The validation was done by comparing the difference between the reactivity evaluated using a truncated measurements, and the reactivity evaluated when taking the full measurement. The analysis was repeated for two models of the Feynman-\(\gamma\) function— the prompt and the delayed reactivity models.

For both the prompt and the delayed reactivity models, results indicate a very good correspondence between the expected statistical uncertainty and the measured deviation (naturally, the results for the prompt reactivity are slightly better, but this is only noticeable in very short measurements). One may argue that the data set used for the validation of the formulas is very limited, since all four signals were obtained in a marginally critical system, less than 300 pcm from criticality. While we certainly agree that a validation in a deeply sub critical setting would be favorable, we feel that the results introduced in the study are useful for two reasons: first, since the parameters of the reactor are typically well known, the formulas introduced in Dubi and Kolin (2016) may be used as a planning tool, and then the statistical uncertainty may validated after the experiment by sampling higher moments in a more traditional fashion (Pacilio, 1965; Jammes et al., 2002; Smith-Nelson et al., 2015).

Second, although taken on a limited data set, we think the results form a strong validation: for the prompt reactivity model, we do not expect to see any real differences (in the present context) between marginally and deeply sub critical settings. In the delayed reactivity model, it has been argued that the linear approximation of the Inhour equation grows tighter as the system becomes more sub critical. Therefore, if anything, we expect that the approximation introduced in Dubi and Kolin (2016) would work even better.

From an operational point of view, the main question this study aims to address is very simple: how long should we measure in order to have a viable result? Clearly, a general answer can not be given, because it depends on both on the system parameters and both on the definition of “viable”. However, when examining the sensitivity of the uncertainty graph - Fig. 1- we notice that the true importance is in the reactivity (and, of course, measurement time), while all the other parameters have a fairly small contribution. Therefore, we can suggest the following measurement times:

1. For marginally critical systems \((\rho = -400)\) pcm to \(\rho = 0\) pcm—20 min.
2. Intermediate reactivity levels \((\rho = -1500)\) pcm to \(\rho = -500\) pcm—45 min to 1 h.
3. Deeply sub-critical systems \((\rho < -1500)\)–2 h.
4. Under \(\rho = -5000\) pcm—extremely long measurements.

Of course, these recommended times should be treated as a guideline, rather than a strict value.

References


